SOLUTION

(a) \( x^3 + x^2 + 4x + 4 = (x^3 + x^2) + (4x + 4) \)
\[= x^2(x + 1) + 4(x + 1) \]
\[= (x^2 + 4)(x + 1) \]

(b) \( x^3 - 2x^2 - 3x + 6 = (x^3 - 2x^2) - (3x - 6) \)
\[= x^2(x - 2) - 3(x - 2) \]
\[= (x^2 - 3)(x - 2) \]

1.3.4

EXERCISES

1-38 = Perform the indicated operations and simplify.
1. \((3x^2 + x + 1) + (2x^2 - 3x - 5)\)
2. \((3x^2 + x + 1) - (2x^2 - 3x - 5)\)
3. \((x^3 + 6x^2 - 4x + 7) - (3x^2 + 2x - 4)\)
4. \(3(x - 1) + 4(x + 2)\)
5. \(8(2x + 5) - 7(x - 9)\)
6. \(4(x^2 - 3x + 5) - 3(x^2 - 2x + 1)\)
7. \(2(2 - 5r) + r^2(t - 1) - (t^4 - 1)\)
8. \(5(3t - 4) - (t^2 + 2) - 2t(t - 3)\)
9. \(\sqrt[3]{x} (x - \sqrt[3]{x})\)
10. \(x^{3/2}(\sqrt{x} - 1/\sqrt{x})\)
11. \(\sqrt[3]{y} (y^2 - 1)\)
12. \((4x - 1)(3x + 7)\)
13. \((3t - 2)(7t - 5)\)
14. \((r + 6)(t + 5) - 3(t + 4)\)
15. \((x + 2y)(3x - y)\)
16. \((4x - 3y)(2x + 5y)\)
17. \((1 - 2y)^2\)
18. \((3x + 4)^2\)
19. \((2x - 5)(x^2 - x + 1)\)
20. \((x^2 + 3)(5x - 6)\)
21. \(x(x - 1)(x + 2)\)
22. \((1 + 2x)(x^2 - 3x + 1)\)
23. \((2x^2 + 3y^2)^2\)
24. \((x^{1/2} + y^{1/2})(x^{1/2} - y^{1/2})\)
25. \((x^2 - a^2)(x^2 + a^2)\)
26. \((\sqrt{h^2 + 1} + 1)(\sqrt{h^2 + 1} - 1)\)
27. \((1 + a)^3\)
28. \((x - 1)(x^2 + x + 1)\)
29. \((\sqrt{a} - \frac{1}{b})(\sqrt{a} + \frac{1}{b})\)
30. \((c + \frac{1}{c})^2\)
31. \((x^2 + x - 2)(x^3 - x + 1)\)

32. \((1 + x + x^2)(1 - x + x^2)\)
33. \((1 + x^{1/2})(1 - x^{1/2})\)
34. \((x^{3/2} - x + 1)(x^2 + x^{1/2} - 2)\)
35. \((1 - b)^2(1 + b)^2\)
36. \((1 + x - x^2)^2\)
37. \((3x^2y + 7xy^3)(x^2y^3 - 2y^2)\)
38. \((x^4y - y^5)(x^2 + xy + y^2)\)

39-88 = Factor the expression completely.
39. \(2x + 12x^3\)
40. \(8x^2 + 4x^3\)
41. \(6y^4 - 15y^3\)
42. \(5ab - 8abc\)
43. \(x^3 + 7x + 6\)
44. \(x^3 - x - 6\)
45. \(x^4 - 2x - 8\)
46. \(x^2 - 14x + 48\)
47. \(y^2 - 8y + 15\)
48. \(y^2 + 2y - 16\)
49. \(2x^2 + 5x + 3\)
50. \(2x^2 + 7x - 4\)
51. \(9x^2 - 36\)
52. \(8x^2 + 10x + 3\)
53. \(6x^2 - 5x - 6\)
54. \(6 + 5r - 6r^2\)
55. \((x - 1)(x + 2)^2 - (x - 1)^2(x + 2)\)
56. \((x + 1)^3x - 2(x + 1)^2x^2 + x^3(x + 1)\)
57. \(y^3(y + 2)^3 + y^3(y + 2)^4\)
58. \(n(x - y) + (n - 1)(y - x)\)
59. \((a - 1)b^2 - 4(a - 1)^2\)
60. \((a + b)^2 - (a - b)^2\)
61. \(r^2 + 1\)
62. \(4r^2 - 9s^2\)
63. \(4r^2 - 12r + 9\)
64. \(x^3 - 27\)
65. \(x^3 + 2x^2 + x\)  
66. \(3x^3 - 27x\)  
67. \(4x^2 + 4xy + y^2\)  
68. \(4r^2 - 12rs + 9s^2\)  
69. \(x^4 + 2x^3 - 3x^2\)  
70. \(x^6 + 64\)  
71. \(8x^3 - 125\)  
72. \(x^4 + 2x^2 + 1\)  
73. \(x^4 + x^2 - 2\)  
74. \(x^3 + 3x^2 - x - 3\)  
75. \(y^3 - 3y^2 - 4y + 12\)  
76. \(y^3 - y^2 + y - 1\)  
77. \(2x^3 + 4x^2 + x + 2\)  
78. \(3x^3 + 5x^2 - 6x - 10\)  
79. \((3 + x)^2 - (1 + x)^2\)  
80. \((1 + \frac{1}{x})^2 - \left(1 - \frac{1}{x}\right)^2\)  
81. \(x^{5/2} - x^{1/2}\)  
82. \(3x^{3/2} + 4x^{1/2} + x^{3/2}\)  
83. \(x^{-3/2} + 2x^{-1/2} + x^{1/2}\)  
84. \((x - 1)^{3/2} - (x - 1)^{1/2}\)  
85. \((x + 1)^{1/2} + 2(x^2 + 1)^{1/2}\)  
86. \(x^{-1/2}(x + 1)^{1/2} + x^{1/2}(x + 1)^{-1/2}\)  
87. \((a^2 + 1)^2 - 7(a^2 + 1) + 10\)  
88. \((a^2 + 2a)^2 - 2(a^2 + 2a) - 3\)  

89-92 # Factor the expression completely. (This type of expression arises in calculus when using the "product rule.")  
89. \(3x^3(4x - 12)^2 + x^4(4x - 12)(4)\)  
90. \(5(x^2 + 4)^4(2x)(x - 2)^3 + (x^2 + 4)^4(4)(x - 2)^3\)  
91. \(3(2x - 1)^2(2)(x + 3)^{1/2} + (2x - 1)^{(1/2)}(x + 3)^{-1/2}\)  
92. \(\frac{1}{2}(x + 6)^{-2/3}(2x - 3)^2 + (x + 6)^{1/3}(2x - 3)(2)\)  
93. (a) Show that \(ab = \frac{1}{2}[(a + b)^2 - (a^2 + b^2)]\).  
(b) Show that \((a^2 + b^2)^2 - (a^2 - b^2)^2 = 4a^2b^2\).  
(c) Show that \((a^2 + b^2)(c^2 + d^2) = (ac + bd)^2 + (ad - bc)^2\).  
(d) Factor completely: \(4a^2c^2 - (a^2 - b^2 + c^2)^2\).  
(e) Factor \(x^4 + 3x^2 + 4\). [Hint: Write the expression as \((x^2 + 4x^2 + 4) - x^2\) and use the Difference of Squares Formula.]  

94. Verify each formula algebraically.  
(a) Special Product Formulas 1 and 2  
(b) Special Product Formulas 3 and 4  
(c) Factoring Formula 4  
(d) Factoring Formula 5

**DISCOVERY • DISCUSSION**

95. The Power of Algebraic Formulas. Use the Difference of Squares Formula to factor \(17^2 - 16^2\). Notice that it is easy to calculate the factored form in your head, but not so easy to calculate the original form in this way. Evaluate each expression in your head:  
(a) \(52^2 - 527^2\)  
(b) \(122^2 - 120^2\)  
(c) \(1020^2 - 1010^2\)  
Now use Special Product Formula 1 to evaluate these products in your head:  
(d) \(49 \cdot 51\)  
(e) \(998 \cdot 1002\)  
96. Volume of a Cylindrical Shell. Using the formula for the volume of a cylinder given on the inside back cover of this book, explain why the volume of the cylindrical shell shown in the figure is  
\[V = \pi R^2 h - \pi r^2 h\]

Factor this to show that  
\[V = 2\pi \cdot \text{average radius} \cdot \text{height} \cdot \text{thickness}\]  
Use the "unrolled" diagram to explain why this makes sense geometrically.

97. Differences of Even Powers.  
(a) Factor the expressions completely: \(A^4 - B^4\) and \(A^8 - B^8\).  
(b) Verify that \(18,335 = 12^4 - 7^4\) and that \(2,868,335 = 12^8 - 7^8\).  
(c) Use the results of parts (a) and (b) to factor the integers \(18,335\) and \(2,868,335\). Show that in both of these factorizations, all the factors are prime numbers.

98. Factoring \(A^n - 1\). Verify the factoring formulas in the list by expanding and simplifying the right-hand side in each case:  
\[A^2 - 1 = (A - 1)(A + 1)\]  
\[A^3 - 1 = (A - 1)(A^2 + A + 1)\]  
\[A^4 - 1 = (A - 1)(A^3 + A^2 + A + 1)\]  
Based on the pattern displayed in this list, how do you think \(A^2 - 1\) would factor? Verify your conjecture. Now generalize the pattern you have observed to obtain a factorization formula for \(A^n - 1\), where \(n\) is a positive integer.